VISUALIZING AND UNDERSTANDING COMPLEX SYSTEMS

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Plan

1. Introduction: Complex systems and Complex networks

2. Case studies:
   - Public transport networks
   - Online game networks
   - Networks of ancient narratives
Complex systems
of many interacting agents
Complex systems

The study of complex systems represents a new approach to science that investigates how relationships between parts give rise to the collective behaviors of a system... The equations from which models of complex systems are developed generally derive from statistical physics, information theory and non-linear dynamics.
The whole is greater than the sum of its parts.
Aristotle

More is different.
Philip Anderson

Complex systems science bridges the natural and social sciences, enriching both, and reduces the gap between science, engineering, and policy.
CSS Website

In recent years physicists have been deeply interested in studying the behavior of complex systems. The result of this effort has been a conceptual revolution, a paradigmatic shift that has far reaching consequences for the very definition of physics.
Giorgio Parisi

I think the next century will be the century of complexity.
Stephen Hawking
Complex systems inherent features

- **SELF-ORGANIZATION** [into patterns, as occurs with flocks of birds, periodicity in disease outbreaks, or residential segregation.]

- **EMERGENCE** [of functionalities, such as cognition in the brain or the robustness of networks.]

- **CHAOS** [where small changes in initial conditions ("the flapping of a butterfly's wings in Argentina") produce large later changes ("a hurricane in the Caribbean").]

- **"FAT-TAIL" BEHAVIOR** [where rare events occur much more often than would be predicted by a normal (bell-curve) distribution.]

- **ADAPTIVE INTERACTION** [where interacting agents (as in markets or the Prisoner's Dilemma) modify their strategies in diverse ways as experience accumulates to produce cooperative behavior.]
Complex networks
as a tool to visualize, quantify and understand complex systems
Complex networks

A graph: vertices edges

a set of • connected via ——
Complex networks

A network: nodes links
Complex networks

A graph: vertices edges

A network: nodes links

\[ N = 4 \]

\[ k_1 = k_2 = k_3 = 3, \]

\[ k_4 = 5 \]

\[ N \sim 10^3 - 10^9 \]

\[ P(k), \langle k \rangle, \ldots \]
Network features: distance, correlations, centrality

- Mean shortest path length:
  \[ \langle l \rangle = \frac{2}{N(N-1)} \sum_{i>j} l_{ij} \]

- Clustering coefficient:
  \[ C_m = \frac{2E_m}{k_m(k_m-1)} \]

- Betweenness:
  \[ \sigma(m) = \sum_{i \neq j} \frac{B(i,m,j)}{B(i,j)} \]
Network features: Small worlds

Watts, Strogats’98:

$N$ vertices of degree $k$
rewiring probability $p$

$N = 20$
$k = 4$

Regular

\[
\langle l \rangle \sim \frac{N}{2k} \gg 1
\]

\[
C \sim \frac{3}{4}
\]

Small-world

\[
\langle l \rangle \sim \langle l \rangle_{\text{rand}}
\]

\[
C \gg C_{\text{rand}}
\]

Random

\[
\langle l \rangle_{\text{rand}} \sim \frac{\ln N}{\ln \langle k \rangle}
\]

\[
C_{\text{rand}} \sim \frac{\langle k \rangle}{N} \ll 1
\]
self-organization
emergence of functionalities
fat-tail behaviour
adaptive interaction

Public transport networks
Online game networks
Networks of ancient narratives
In collaboration with:

Bertrand Berche
Christian von Ferber
Taras Holovatch
Vasyl Palchykov
Robin de Regt

Supported by:

IRSES N269139 (DCP-PhysBio); N295302 (SPIDER)
Our database

Number of Routes

Number of stations

Los Angeles

Moscow

Sydney

London

Sao Paolo

Rome

Istanbul

Hamburg

Taipei

Hong Kong

Paris

Berlin

Dallas
Network interpretation

Public transit map

(Bipartite) $B$-space

$L$-space

$P$-space

$C$-space
Robustness vs vulnerability

Largest cluster size $S$ as function of removed node share $c$.

Scenario "random".

Robustness vs vulnerability

Largest cluster size $S$ as function of removed node share $c$.

Scenario "recalculated node degree".

Transportation network seen as a fractal

Fractal dimension of the UK coach network calculated by considering a boxing method

In collaboration with:

Benedikt Fuchs
Olesya Mryglod
Michael Szell
Stefan Thurner

Online game networks

Supported by:

IRSES N612707 (DIONICOS); COST TD1210 (KNOWSCAPE)
Sociophysics. Analysis of a virtual world

MMOG (online game, launched in 2004, ∼ 400 000 players)

Friendly (green) and hostile (red) relations between 72 randomly chosen players, 445-th day

M. Szell, S. Thurner, Social Networks 32 (2010) 313
Quantitative features of PARDUS universe

- M.S. Granovetter’73, *The Strength of Weak Ties*
  Overlap $O$ of two neighbouring nodes as function of weight $w$ and betweenness $b$ of their connecting link:
  \[ O(w) \simeq 3^{\frac{w}{2}} \quad O(b) \simeq \frac{1}{\sqrt{b}} \]

- R. Dunbar’93, *Co-Evolution of Neocortex Size, Group Size and Language in Humans*
  Maximal node degree (Dunbar number):
  \[ k_{\text{max}} \simeq 150 \]

*M. Szell, S. Thurner, Social Networks 32 (2010) 313*
Power laws and universality

Social dynamics: interevent time distribution $P \sim \tau^{-2}$

Self-organization and chaos

Extreme event statistics: wars in Pardus universe

Emergence of structures in social networks

Action streams in virtual world
In collaboration with:

Ralph Kenna

Pádraig Mac Carron

Petro Sarkanych

Networks of ancient narratives

Supported by:

IRSES N269139 (DCP-PhysBio); N295302 (SPIDER)
The Cattle Raid of Cooley

Táin Bó Cúailnge (The Tain): the most famous epics of Irish mythology

Network of friends

Social network of bylny characters

Hostile (red) and friendly (blue) bylny characters networks of Kievan period

Universal properties of myth networks

Node degree distribution (The Tain)

## Universal properties of myth networks

<table>
<thead>
<tr>
<th>Property</th>
<th>Social</th>
<th>Myth (friendly)</th>
<th>Fiction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small world</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Hierarchy</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$p(k)$</td>
<td>Power law</td>
<td>Power law</td>
<td>Exp.</td>
</tr>
<tr>
<td>Scale free</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>$G_c$</td>
<td>&lt; 90%</td>
<td>&lt; 90%</td>
<td>&gt; 90%</td>
</tr>
<tr>
<td>TA</td>
<td>Vulnerable</td>
<td>Vulnerable</td>
<td>Robust</td>
</tr>
<tr>
<td>RA</td>
<td>Robust</td>
<td>Robust</td>
<td>Robust</td>
</tr>
<tr>
<td>Assortative</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Complex systems of many interacting agents can be visualized, quantified and understood by tools of Complex networks.

- self-organization
- phase transition
- emergence of functionalities
- percolation
- fat-tail behaviour
- scaling, universality
- adaptive interaction
- diffusion, RW, SAW